NC STATE UNIVERSITY

Noisy Quantum Computing

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NC STATE UNIVERSITY

Outline

- Revisit some of the physics postulates of quantum mechanics and concepts of a Bloch sphere
- Investigate the question as to how a quantum computing system evolves from a given initial state
- Calculate some properties and characteristics of such systems with several examples

Impacts of “Time Evolution” on a System from Perspective of the Postulates of Quantum Mechanics

Postulate 5

5. The operator $\mathbb{A}$ corresponding to an observable that yields a measured value "a_n" will correspond to the state of the system as the normalized eigenstate $|a_n>$

Postulate 5 Implications for Quantum Computing

- This postulate describes the collapse of the wave packet of probability amplitudes when making a measurement on the system
- A system described by a wave packet $|\Psi >$ and measured by an operator $\mathbb{A}$ repeated times will yield a variety of results given by the probabilities $|<a_n|\,\Psi >|^2$
- If many identically prepared systems are measured each described by the state |a> then the expectation value of the outcomes is

$$< a > \equiv \sum_n a_n \, \mathrm{Prob}(a_n) =< a|\mathbb{A}|a>$$

Postulate 6

Dynamics - Time Evolution of a Quantum Mechanical System

- The evolution of a closed system that evolves over time is expressed mathematically by a unitary operator that connects the system between time t_1 to time t_2 and that only depends on the times t_1 and t_2

- The time evolution of the state of a closed quantum system is described by the Schrodinger equation

$$i\hbar \frac{d}{dt} |\Psi >= H(\mathrm{t})|\Psi >$$

Postulate 6 Implications for Quantum Computing

- Any type of “program” that would represent a step by step evolution from an initial state on a quantum computer to some final state must preserve the norm of the state (conservation of probability)

- Requirement that each “step-by-step” evolution must preserve unitarity (forces constraints for “programming” a quantum computer)

- The requirement of postulate 6 that the quantum mechanical system be closed for this unitary evolution of the system over time (forces constraints for “programming” a quantum computer)

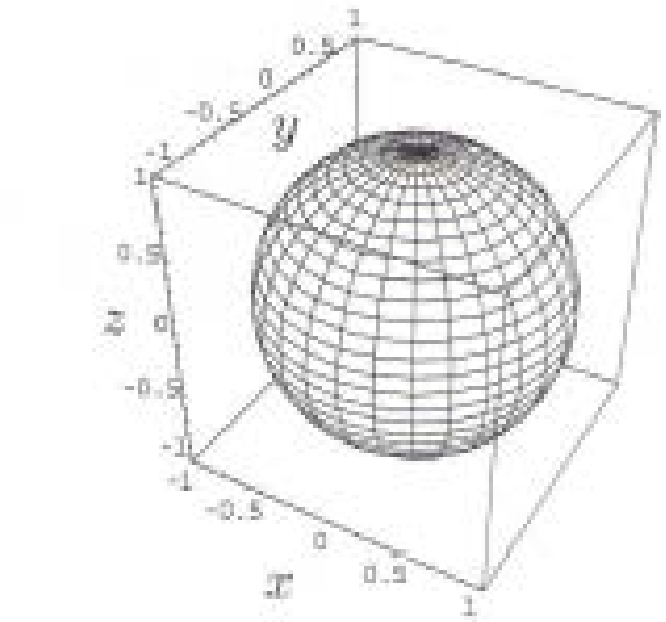
Evolution of a Quantum System

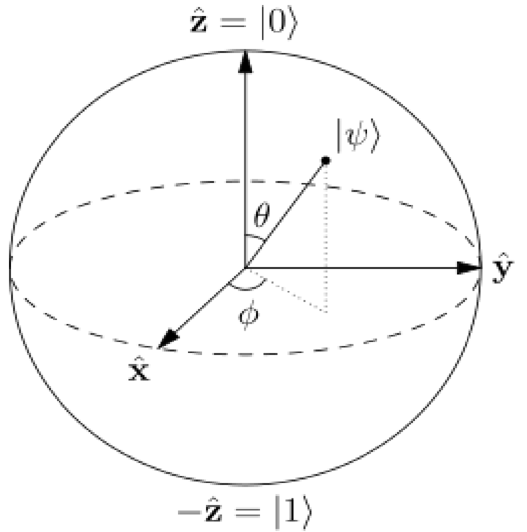
- Physics postulate 5 describes a the process for obtaining a measurement from a quantum mechanical state
- Postulate 6 describes the time evolution of a quantum mechanical system
- It is known that quantum information is extremely fragile due to interactions between the system and the overall environment in which that system is embedded

Bloch Sphere

- The matrices σ_x, σ_y and σ_z are associated with rotations about the x, y, and z axes

$$R_{\hat{n}}(\theta) \equiv e^{-i\theta \hat{n}\cdot\frac{\sigma}{2}} = \cos(\frac{\theta}{2})I - i\sin(\frac{\theta}{2})(n_x X + n_y Y + n_z Z))$$





- Reversible one qubit gates can be viewed as rotations in this 3 dimensional representation

How Will Qubits Evolve under Unitary Transformations

- How will quantum system evolve in time from the initial state at t=0 to a later time t = T in the presence of the environment in which hit is embedded

- What mathematical relationships and metrics can be defined to describe this process

Evolution of a QM System Described by a Hamiltonian

- Physics postulate 6 describes the quantum mechanical evolution of that system

$$ih\frac{d}{dt}|\Psi >= H(\mathrm{t})|\Psi >$$

- That pure state will co-evolve with the degrees of freedom to which it couples
- Significant degrees of freedom to which it couples are the environment

Hamiltonian for a QM Evolving System

- Define H_S to be the system Hamiltonian, H_E the environment Hamiltonian, and H_{SE} to be the interaction Hamiltonian such that
$$H_T = H_S + H_E + H_{SE}$$
- Assuming that the system environment Hamiltonian is negligible the Hilbert space describing the system and environment are
$$\mathcal{H}_T = \mathcal{H}_S \otimes \mathcal{H}_E$$
- A system evolves by having a unitary operator U act on the initial state
$$|\Psi > \mapsto U|\Psi >$$
- <u>Although the total time evolution of the system and environment can be assumed to be unitary, the question of the evolution restricted to the system state generally is not unitary</u>

Density Matrices

- Introduce a density operator described by a density matrix
- Pure states have very simple density matrices that can be written as

$$\rho_{\Psi} = |\Psi >< \Psi|$$

- Assuming that the initial state is described by a pure state

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Quantum Errors

- This loss of unitarity during time evolution can be attributed to two basic types of quantum gate errors
  1. Coherent errors – these are errors that preserve the purity of the input states based on a perturbed unitary operation $(U' \neq U)$
  $$|\Psi > \mapsto U'|\Psi >$$
  2. Incoherent errors – those that do not preserve the purity of the input state
- Incoherent errors must be represented in terms of density matrices and an evolution operator

$$\rho \mapsto \sum_{j=1}^{n} K_j \rho K_j^{\dagger}$$

What are these operators K ?

The Evolution of a Quantum System Under Unitary Transformations

- The state of a total system can described by a density matrix ρ that evolves from an initial state $\rho(0) = \rho_S \otimes \rho_E$
- Label ρ_S as the initial density matrix of the system of interest and ρ_E as the density matrix representing the environment
- The total system is assumed to be closed and evolve with a unitary matrix $U(t)$ as $\rho(t) = U(t)(\rho_S \otimes \rho_E)U(t)^\dagger$
- The goal is to extract information on the state of the system of interest at some later time $t > 0$
- However even under these restricted conditions the resulting evolved state is not a tensor product state in general

Tracking the Evolution of the System

- Assuming that the initial state ρ_0 is a pure state at $t = 0$ it cannot be assumed that at a time $t > 0$ pure pure state information can be extracted from the trace of the density matrix $\rho(t)$
- Nevertheless it is still possible to define the system density matrix $\rho_S(t)$ by taking the Trace over the environment
- $\rho_S(t) = Tr_E[U(t)(\rho_S \otimes \rho_E)U(t)^\dagger]$

Tracking the Evolution of the System (contd́)

- Construct a basis for H_T where $|e_j>$ represents the system and $|\epsilon_a>$ represents the environment
- The initial density matrices are written as

$$\rho_S = \sum_j p_j |e_j >< e_j| \tag{1}$$

$$\rho_E = \sum_a r_a |\epsilon_a >< \epsilon_a| \tag{2}$$

Tracking the Evolution of the System (contd́)

- Time evolution operator on the basis for H_T is $U(T)|e_j, \epsilon_a >$
- The density matrix can now be written as

$$U(t)(\rho_S \otimes \rho_E)U(t)^\dagger = \sum_{j,a} p_j r_a U(t)|e_j, \epsilon_a >< e_j, \epsilon_a|U(t)^\dagger \quad (3)$$

$$= \sum_{j,a,k,b,l,c} p_j r_a U_{kb;ja}|e_k, \epsilon_b >< e_l, \epsilon_c|U^*_{lc;ja} \quad (4)$$

Partial Trace

- Calculate the partial Trace over H_E

$$\rho_S(t) = Tr_E[U(t)(\rho_S \otimes \rho_E)U(t)^\dagger] \quad (5)$$

$$= \sum_{j,a,b} p_j(\sum_k \sqrt{r_a} U_{kb;ja}|e_k >)(\sum_l \sqrt{r_a} < e_l|U_{kb;ja}) \quad (6)$$

- Assume the the environment can be initialized in a pure state $\rho_E = |\epsilon_0 >< \epsilon_0|$
- From this assumption $\rho_S(t)$ can be expressed in closed form

$$\rho_S(t) Tr_E[U(t)(\rho_S \otimes |\epsilon_0 >< \epsilon_0|)U(t)^\dagger] \quad (7)$$

$$= \sum_a (\mathcal{I} \otimes (\epsilon_a|)U(t)(\rho_S \otimes |\epsilon_0 >< \epsilon|_0)|U(t)^\dagger(\mathcal{I} \otimes |\epsilon_a)) \quad (8)$$

$$= \sum_a (\mathcal{I} \otimes (\epsilon_a|)U(t)(\mathcal{I} \otimes |\epsilon_0 >)\rho_S(\mathcal{I} \otimes < \epsilon_0|)U(t)^\dagger(\mathcal{I} \otimes |\epsilon_a)) \quad (9)$$

Kraus Operators

- Define a **Kraus operator** $E_a(t) : H_S \to H_S$ and the operator-sum representation of a quantum operator $\mathcal{E}$
- It should be noted that because we are working with a closed system the completeness and trace-preserving properties are satisfied $(1 = Tr_S \rho_S(t) = Tr_S(\sum_a E_a^\dagger E_a \rho_S)$

$$E_a(t) = < \epsilon_a | U(t) | \epsilon_0 > \tag{10}$$

$$\mathcal{E} = \rho_S(t) \sum_a E_a(t) \rho_S E_a(t)^\dagger \tag{11}$$

This E_a term corresponds to what was earlier written as K_j

Generalization of the Unitary Operator U

- Based on the statement of Postulate 6 we have considered time evolution unitary operators that act on a system
- There is a generalization to this idea of a unitary transformation that is not constrained to only time dependent evolution
- Let U be any type of operation that can be expressed as a *black box* unitary operator

Generalization of the Unitary Operator U - CNOT Example

- Consider the general example of a CNOT gate where the control bit is defined as the system of interest and the target bit is the environment
- Using the operator sum representation can write each term E_a

$$E_0 = P_0 = (\mathcal{I} \otimes < 0|) U_{CNOT} (\mathcal{I} \otimes |0 >) \quad (12)$$

$$E_1 = P_1 = (\mathcal{I} \otimes < 1|) U_{CNOT} (\mathcal{I} \otimes |0 > \quad (13)$$

$$\mathcal{E} = P_0 \rho_S P_0 + P_1 \rho_S P_1 = \rho_{00} P_0 + \rho_{11} P_1 = \begin{pmatrix} \rho_{00} & 0 \\ 0 & \rho_{11} \end{pmatrix}$$

$$where \rho_S = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

Comments and Observations

- Quantum operations do not necessarily map density matrices to density matrices associated with the same state space
- Tracing out the extra degrees of freedom makes it impossible to invert a quantum operation
- Essentially this means that if a system starts in an initial configuration described by a density matrix ρ_S there are infinitely many unitarty operators U that will yield the same $\mathcal{E}(\rho_S)$
- Mathematically the set of operations is no longer a group

Examples of Quantum Computing Processes That Display Decoherence

- Bit-Flip
- Phase Flip
- Depolarization
- Amplitude-Damping

Input Qubit State Represented on a Bloch Sphere*

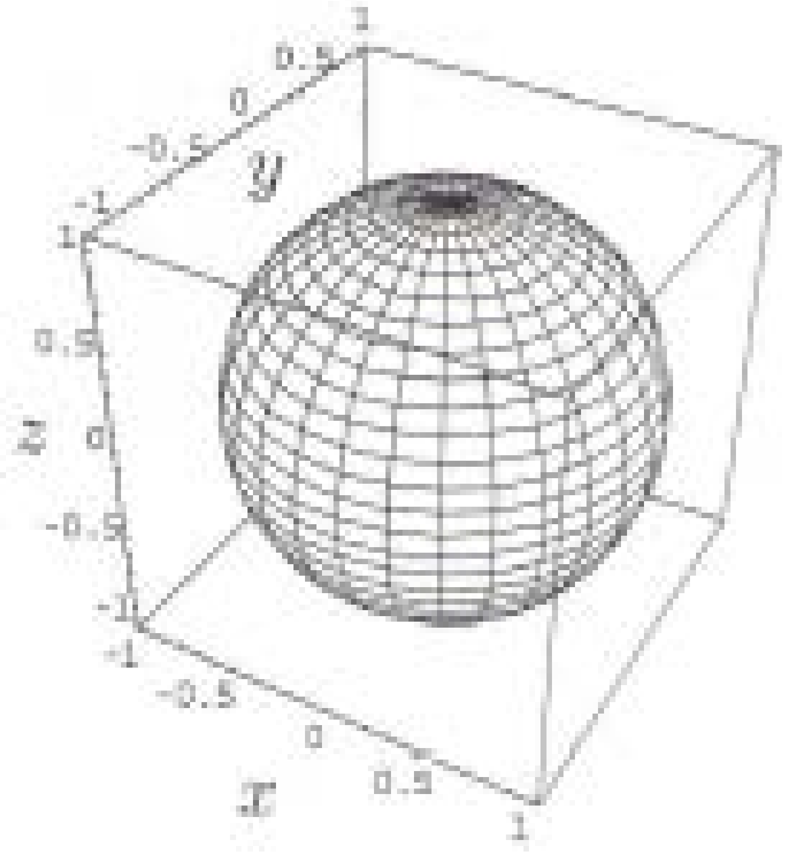


*Figure from Quantum Computing
Nakahara and Ohmi

Bit-Flip Channel

- Defined by a quantum operation
- $\mathcal{E}(\rho_s) = (1-p)\rho_s + p\sigma_x\rho_s\sigma_x$, $0 \leq \text{p} \leq 1$
- the input ρs is bit-flipped $|0> \rightarrow |1>$ and $|1> \rightarrow |0>$ with a probability "p" while it remains in its input state with a probability "(1-p)"
- Can read off the Kraus operators from $\mathcal{E}(\rho_s)$ as

$$E_0 = \sqrt{1-p} I \tag{14}$$
$$E_1 = \sqrt{p}\sigma_x \tag{15}$$

Impacts of This Transformation on ρ_S

- Parameterize ρ_S using Bloch vector and put into expression for $\mathcal{E}(\rho_S)$

$$\rho_S = \frac{1}{2}(\mathcal{I} + \sum_{k=x,y,z} c_k \sigma_k) \tag{18}$$

$$E(\rho_S) = (1-p)\rho_S + p\sigma_x \rho_S \sigma_x \tag{19}$$

$$= \frac{1-p}{2}(\mathcal{I} + c_x\sigma x + c_y\sigma y + c_z\sigma z) + \frac{p}{2}(\mathcal{I} + c_x\sigma x - c_y\sigma y - c_z\sigma z) \tag{20}$$

$$= \frac{1}{2}\begin{pmatrix} 1+(1-2p)c_x & c_x - i(1-2p)c_y \\ c_x + i(1-2p)c_y & 1-(1-2p)c_z \end{pmatrix}$$

Impact of These Transformation on the Bit-Flip Channel

- The above equation produced a mixture of Bloch vector states $(c_x c_y c_z)$ and $(c_x - c_y - c_z)$ with weights (1-p) and p
- Radius of the Bloch sphere is reduced along the y and z axes so that the radius in these directions becomes $|1 - 2p|$

Remarks on the Impact of These Transformation on the Bit-Flip Channel

- The above equation produced a mixture of Bloch vector states $(c_x c_y c_z)$ and $(c_x - c_y - c_z)$ with weights (1-p) and p
- Radius of the Bloch sphere is reduced along the y and z axes so that the radius in these directions becomes $|1 - 2p|$

Circuit Model for a Bit-Flip Channel

- This circuit is an inverted CNOT gate
  $V = \mathcal{I} \otimes |0><0| + \sigma_x \otimes |1><1|$
- The output of this circuit is

$$V(\rho_S \otimes [(1-p)|0><0| + p|1><1|])V^\dagger \tag{16}$$
$$= (1-p)\rho_S \otimes |0><0| + p\sigma_x\rho_S\sigma_x|1><1| \tag{17}$$

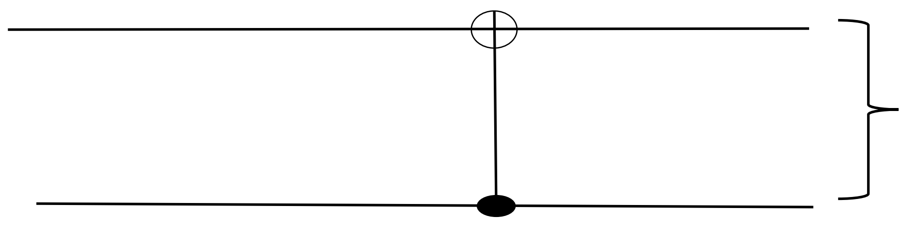
- From the above equation can read off the Kraus operators after tracing over the environmental Hilbert space
- $\mathcal{E}(\rho_S) = (1-p)\rho_S + p\sigma_x\rho_S\sigma_x$

Circuit Model for a Bit Flip Channel

Design a quantum circuit that models this channel – inverted CNOT gate

$\rho_{\{S\}}$

$(1\text{-}p)|0\rangle\langle 0|+p|1\rangle\langle 1|$



$(1\text{-}p)\rho_S \otimes |0\rangle\langle 0| +$
$pX\,\rho_S\,X \otimes |1\rangle\langle 1|$

Bit-Flip Channel
Bloch Sphere* Contracted Along the y and z Axes

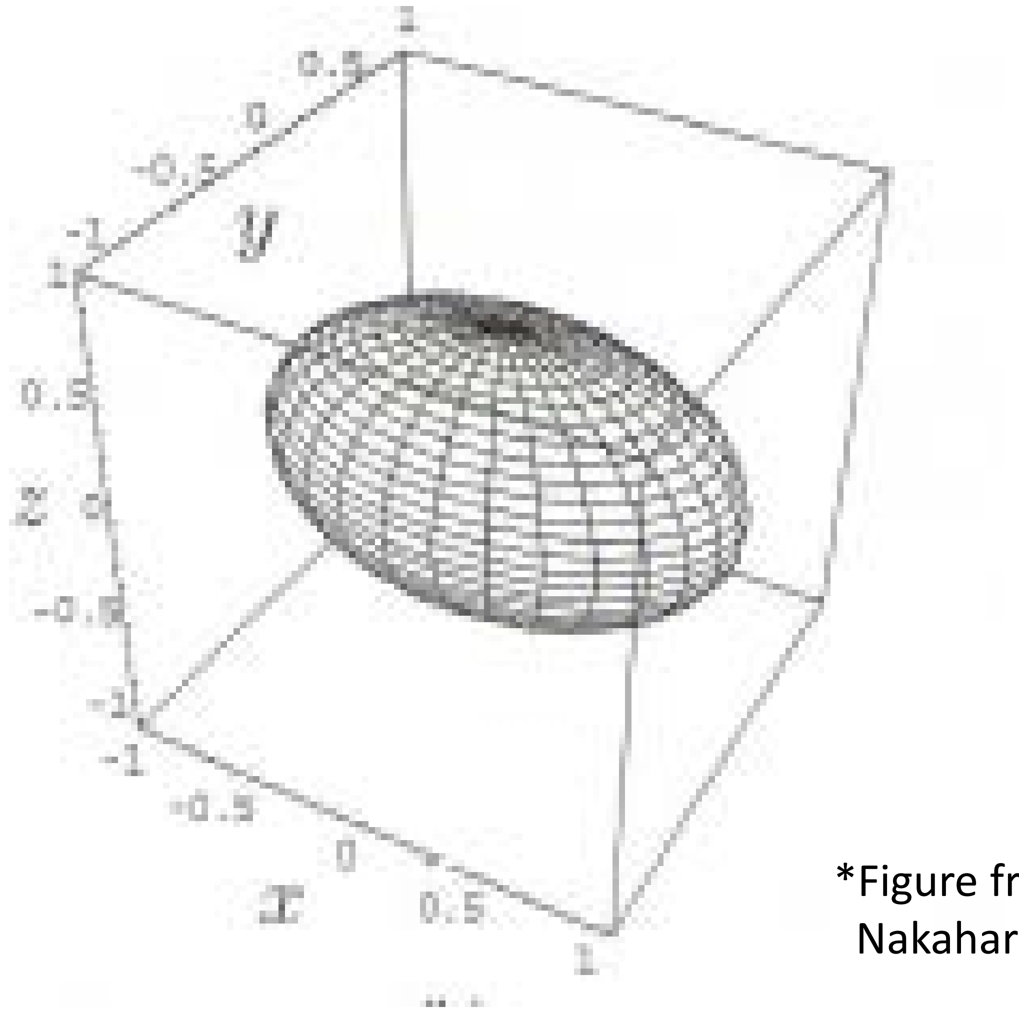


*Figure from Quantum Computing
Nakahara and Ohmi

Phase-Flip Channel

- Defined by a quantum operation
- $\mathcal{E}(\rho_s) = (1-p)\rho_s + p\sigma_z\rho_s\sigma_z$, $0 \leq$ p ≤ 1
- the input ρ_S is phase-flipped $|0> \rightarrow |0>$ and $|1> \rightarrow -|1>$ with a probability "p" while it remains in its input state with a probability "(1-p)"

Kraus operators are

$$E_0 = \sqrt{1-p}\mathcal{I} \tag{21}$$
$$E_1 = \sqrt{p}\sigma_x \tag{22}$$

Phase-Flip Channel

- Parameterize ρ_S using Bloch vector and put into expression for $\mathcal{E}(\rho_S)$

$$\rho_S = \frac{1}{2}(\mathcal{I} + \sum_{k=x,y,z} c_k \sigma_k) \tag{25}$$

$$E(\rho_S) = (1-p)\rho_S + p\sigma_z \rho_S \sigma_z \tag{26}$$

$$= \frac{1-p}{2}(\mathcal{I} + c_x\sigma x + c_y \sigma y + c_z \sigma z) + \frac{p}{2}(\mathcal{I} - c_x \sigma x - c_y \sigma y + c_z \sigma z) \tag{27}$$

$$= \frac{1}{2}\begin{pmatrix} 1+c_z & (1-2p)(-c_x - ic_y) \\ (1-2p)(c_x + ic_y) & 1-c_z \end{pmatrix}$$

Remarks About the Phase-Flip Channel

- Note that the off diagonal elements of this 2x2 matrix decay while the diagonal components do not
- Produced mixture of Bloch vector states (c_x, c_y, c_z) and $(-c_x, -c_y, c_z)$ with weights $(1-p)$ and p
- Initial state has phase $\phi = tan^{-1}\frac{c_y}{c_x}$
- After the quantum operation is applied there is a mixture of ϕ and $\phi + \pi$ states
- This is called a phase relaxation process

Circuit Model for a Phase-Flip Channel

- This circuit is an inverted-z gate $V = \mathcal{I} \otimes |0><0| + \sigma_z \otimes |1><1|$
- The output of this circuit is

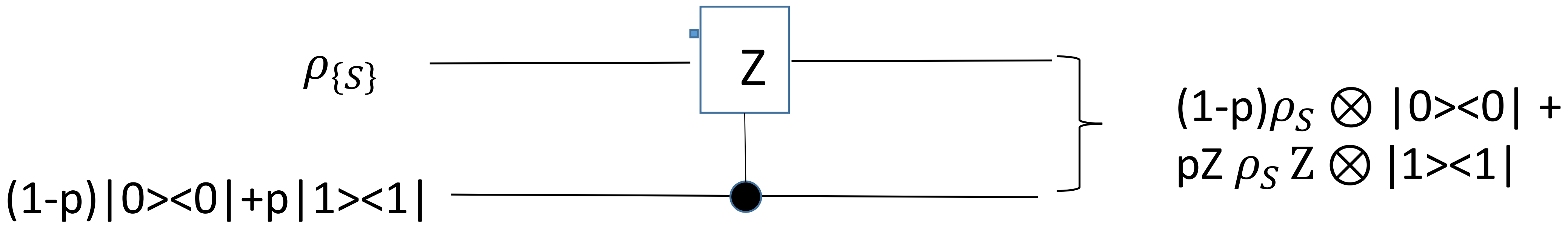
$$V(\rho_S \otimes [(1-p)|0><0| + p|1><1|])V^\dagger \quad (23)$$

$$= (1-p)\rho_S \otimes |0><0| + p\sigma_z\rho_S\sigma_z|1><1| \quad (24)$$

- From the above equation can read off the Kraus operators after tracing over the environmental Hilbert space
- $\mathcal{E}(\rho_S) = (1-p)\rho_S + p\sigma_z\rho_S\sigma_z$

Circuit Model for a Phase Flip Channel

Design a quantum circuit that models this channel – inverted controlled- σ_z gate



Phase-Flip Channel
Bloch Sphere* Contracted Along the x and y Axes

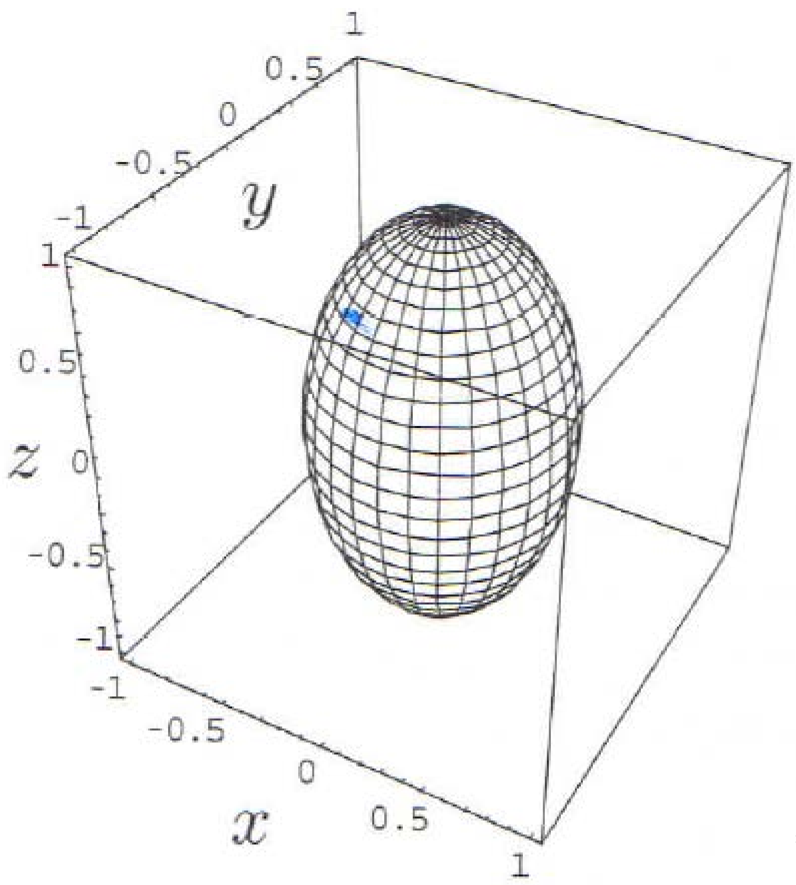


*Figure from Quantum Computing Nakahara and Ohmi

Amplitude Damping Channel

- Describes process where qubit decays from $|1>$ to $|0>$ with probability p
- This is a one way decay process where a qubit **ONLY** decays from $|1>$ to $|0>$ with a probability p
- This downward decay process is mathematically described by a Kraus operator
- $\mathcal{E}(\rho_s) = E_0 \rho_s E_0^\dagger + E_1 \rho_s E_1^\dagger$

Amplitude Damping Channel

This process is mathematically represented by a Kraus operator

$$E_1 = \sqrt{p}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

The Kraus operator for E_0 is fixed by the requirement $\sum_k E_k^\dagger E_k$ must be the identity matrix which yields an expression for E_0

$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}$$

Effect of Amplitude Damping Channel on Bloch Sphere

$$E(\rho_S) = p\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}\rho_S\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix} + \begin{pmatrix}1 & 0\\0 & \sqrt{1-p}\end{pmatrix}\rho_S\begin{pmatrix}1 & 0\\0 & \sqrt{1-p}\end{pmatrix}$$
$$= \begin{pmatrix}\rho_{00} + p\rho_{11} & \sqrt{1-p}\rho_{01}\\ \sqrt{1-p}\rho_{01} & \rho_{11} + p\rho_{11}\end{pmatrix}$$
$$= \frac{1}{2}\begin{pmatrix}1 + [p + (1-p)c_z] & \sqrt{1-p}(c_x - ic_y)\\ \sqrt{1-p}(c_x - ic_y & 1 - [p + (1-p)c_z]\end{pmatrix}$$

Observe

- Observing the components in the 2x2 matrix it can be seen the center of the Bloch sphere is shifted toward the $|0>$ pole by p
- the radius in the x and y directions are reduced by $\sqrt{1-p}$ and the radius in the z direction by (1-p)

Amplitude Damping Channel

1) Bloch Sphere* Shifted Toward the North Pole (|0>) by p
2) Bloch Sphere Radius in x and y Reduced $\sqrt{1-p}$
3) Bloch Sphere Radius in z Reduced 1-p

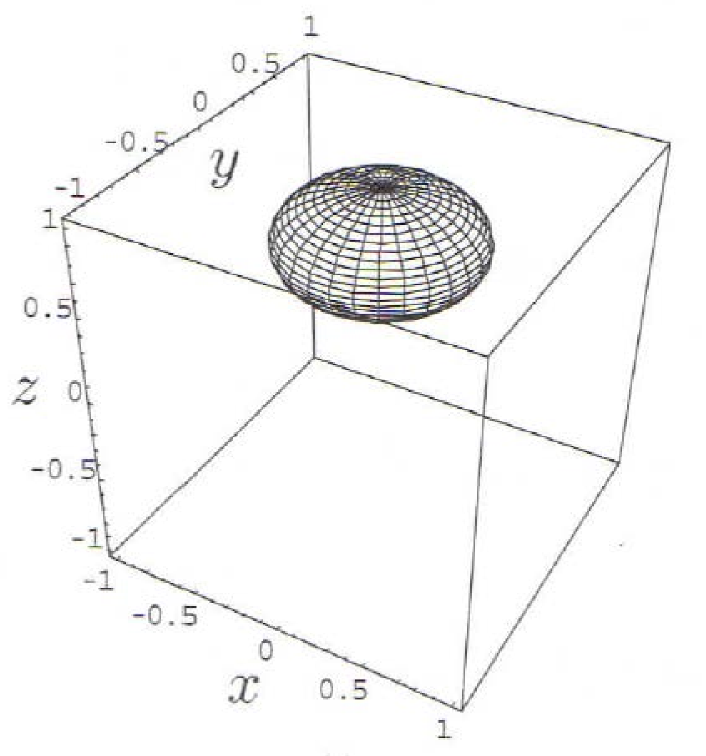


*Figure from Quantum Computing Nakahara and Ohmi

Depolarizing Channel

- A depolarizing channel has the property that it maps the input state ρ to a maximally mixed state with probability p and (1-p)
- $\mathcal{E}(\rho_s) = (1 - p)\rho_s + p\frac{\mathcal{I}}{2}$

Depolarizing Channel (contd́)

- To construct the properties of the depolarizing channel introduce a uniform decomposition equation for the density ρ
- $\rho_S = \frac{1}{2}(\mathcal{I} + c_x\sigma_x + c_y\sigma_y + c_z\sigma_z)$

Writing the in x, y, and z components

$$\sigma_x\rho_S\sigma_x = \frac{1}{2}(\mathcal{I} + c_x\sigma_x - c_y\sigma_y - c_z\sigma_z) \quad (28)$$

$$\sigma_y\rho_S\sigma_y = \frac{1}{2}(\mathcal{I} - c_x\sigma_x + c_y\sigma_y - c_z\sigma_z) \quad (29)$$

$$\sigma_z\rho_S\sigma_z = \frac{1}{2}(\mathcal{I} - c_x\sigma_x - c_y\sigma_y + c_z\sigma_z) \quad (30)$$

This set of equations can be reduced to

$$2\mathcal{I} = \rho_S + \sum_{k=x,y,z} \sigma_k\rho_S\sigma_k \quad (31)$$

Depolarizing Channel (contd́)

Substituting the component equations into $\mathcal{E}$ of the depolarizing channel gives

$$E = (1 - \frac{3}{4}p)\rho_S + \frac{p}{4}\sum_k \sigma_k \rho_S \sigma_k \tag{32}$$

The Kraus operators can now be read off as (k runs over the x, y, z)

$$E_0 = \sqrt{(1 - \frac{3}{4}p)}\mathcal{I} \tag{33}$$

$$E_1 = \sqrt{\frac{p}{4}}\sigma_k \tag{34}$$

Circuit Model for the Depolarizing Channel

- There are 4 Kraus operators which suggests constructing a circuit model of a Fredkin gate with the bottom bit in the diagram being the control bit

The input state can be written as

$$\rho_S \otimes \frac{\mathcal{I}}{2} \otimes [(1-p)|0><0| + p|1><1|] \tag{35}$$

Circuit Model for the Depolarizing Channel

The Fredkin gate acting on this input state yields an output

$$\rho = (\mathcal{I}_4 \otimes |0><0| + U_{SWAP} \otimes |1><1| \tag{36}$$

$$(\rho_S \otimes \frac{\mathcal{I}}{2} \otimes [(1-p)|0><0| + p|1><1| \tag{37}$$

$$(\mathcal{I}_4 \otimes |0><0| + U_{SWAP} \otimes |1><1|) \tag{38}$$

$$= (1-p)\rho_S \otimes \frac{\mathcal{I}}{2} \otimes |0><0| + p\frac{\mathcal{I}}{2} \otimes \rho_S \otimes |1><1| \tag{39}$$

Note that the SWAP gate has the property

$$U_{SWAP}(\rho_1 \otimes \rho_2)U_{SWAP} = (\rho_2 \otimes \rho_1) \tag{40}$$

Tracing the two qubit environment gives

$$Tr_E\rho = (1-p)\rho_S + p\frac{\mathcal{I}}{2} \tag{41}$$

Circuit Model for the Depolarizing Channel

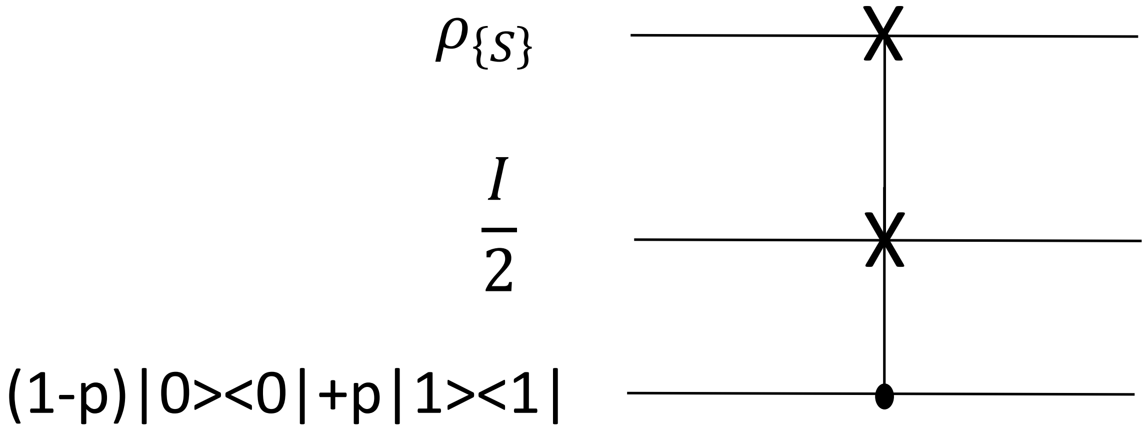
Finally the operator-sum representation for $\mathcal{E}(\rho_S)$ can be written

$$= p\frac{\mathcal{I}}{2} + \frac{1-p}{2}(\mathcal{I} + \sum_k c_k\sigma_k = \frac{\mathcal{I}}{2} + \frac{1-p}{2}\sum_k c_k\sigma_k \qquad (42)$$

Therefore the radius of the Bloch sphere is uniformly reduced from initial size of 1 to (1-p)

Circuit Model for a Depolarizing Channel

Design a quantum circuit that models this channel – inverted Fredkin gate



$$(1\text{-}p)\rho \otimes \frac{I}{2} \otimes |0\rangle\langle 0|$$
$$+\, p\frac{I}{2} \otimes \rho \otimes |1\rangle\langle 1|$$

Depolarizing Channel
Bloch Sphere* Radius Uniformly Reduced from 1 → 1-p

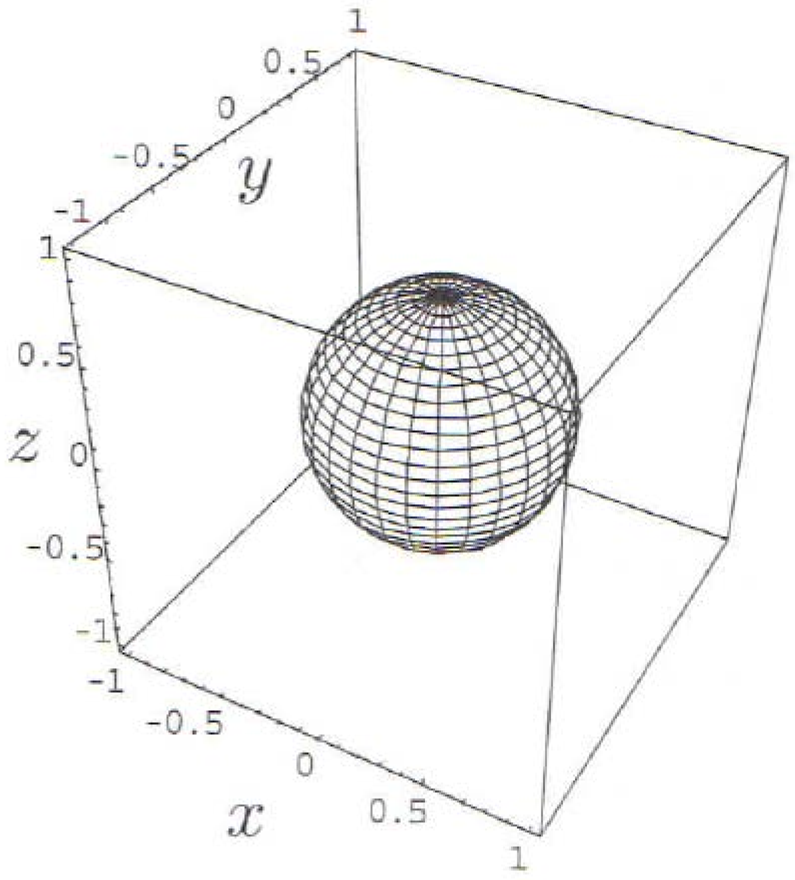
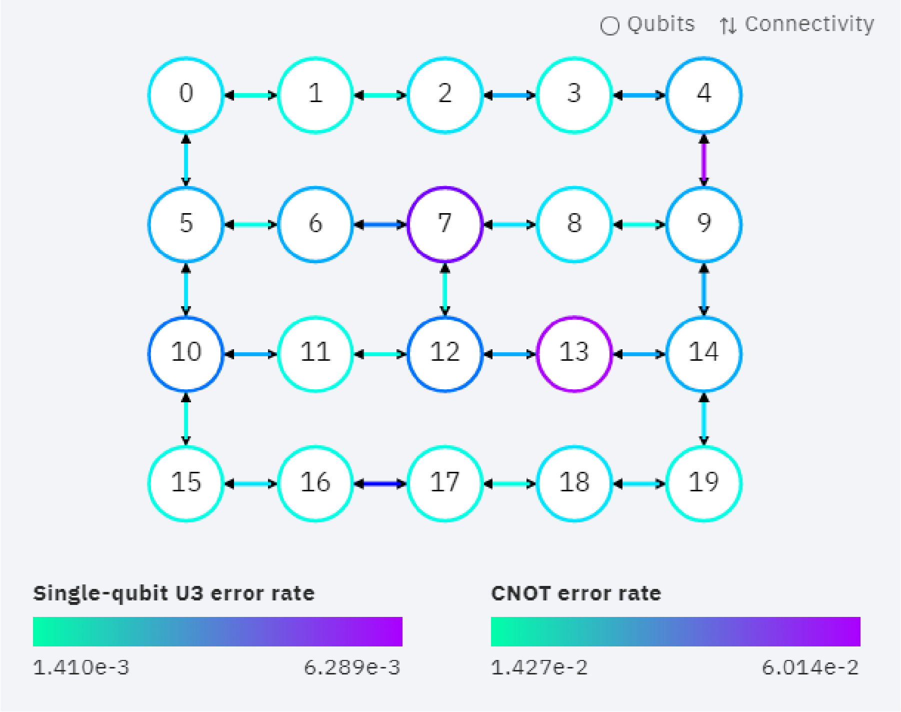


*Figure from Quantum Computing
Nakahara and Ohmi

IBM Quantum Computing Hardware Specific Information

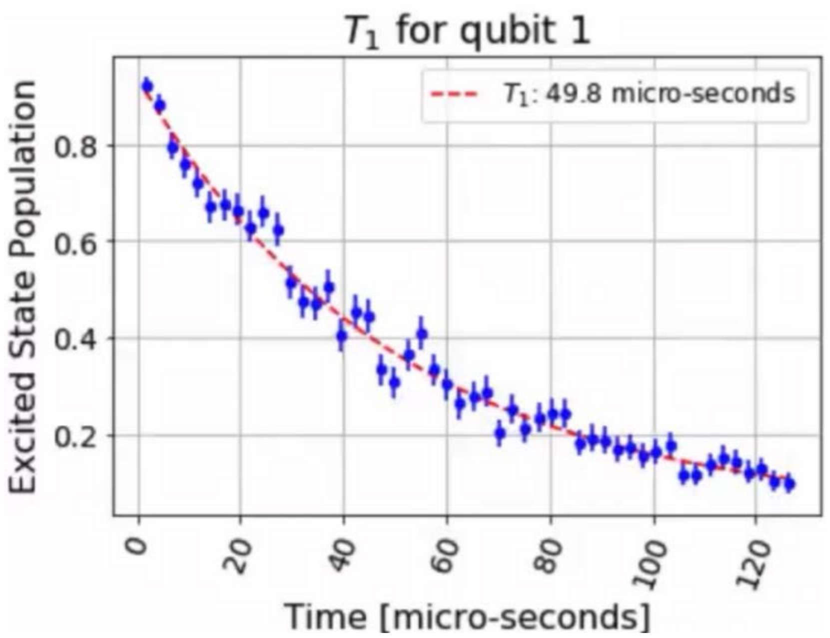
IBM Poughkeepsie



Qubit	T1 (Âµs)	T2 (Âµs)	Frequency (GHz)	Readout error	Single-qubit U3 error rate
Q0	61.69099895	77.98596962	4.919894473	2.10E-02	2.30E-03
Q1	95.01012635	104.764702	4.831965919	2.10E-02	1.80E-03
Q2	48.85106812	61.58236929	4.940458261	4.00E-02	2.45E-03
Q3	95.6229699	98.51223871	4.514750668	2.80E-02	1.73E-03
Q4	73.45983639	65.99098768	4.66291938	6.80E-02	2.51E-03
Q5	75.73526441	67.08900363	4.957352127	4.40E-02	2.50E-03
Q6	73.40514518	96.30681362	4.995568332	2.30E-02	2.84E-03
Q7	80.906214	29.74878872	4.811620184	4.60E-02	5.35E-03
Q8	76.56943421	103.0319559	5.013724514	4.00E-02	2.38E-03
Q9	70.91546525	96.29800297	5.056303512	3.40E-02	2.88E-03
Q10	78.57285481	17.26940926	4.718488921	7.30E-02	3.34E-03
Q11	92.07955089	90.17527103	4.899894597	3.70E-02	1.64E-03
Q12	60.37137943	14.2144111	4.772844284	2.90E-02	3.10E-03
Q13	60.41609818	20.50080243	5.110855608	6.70E-02	6.29E-03
Q14	65.9439697	77.95659012	4.990058848	3.20E-02	2.78E-03
Q15	90.18591429	50.34112158	4.806136481	7.80E-02	1.43E-03
Q16	96.18676005	68.95233031	4.955688421	3.90E-02	1.58E-03
Q17	74.89376096	20.54441189	4.599276481	6.40E-02	1.41E-03
Q18	92.02319589	101.2713455	4.827979719	2.50E-02	2.42E-03
Q19	79.21638543	80.42325778	4.938660492	4.40E-02	1.68E-03

T_1 – Relaxation Time

- Initialize with qubit in the ground state |0>
- Put the qubit into the |1> state by applying an X-gate
- Wait a specified period of time and then measure in the |0> |1> basis
- Find the relaxation rate by fitting and exponential decay curve to the data



IBM figure from Device Characteristics presentation

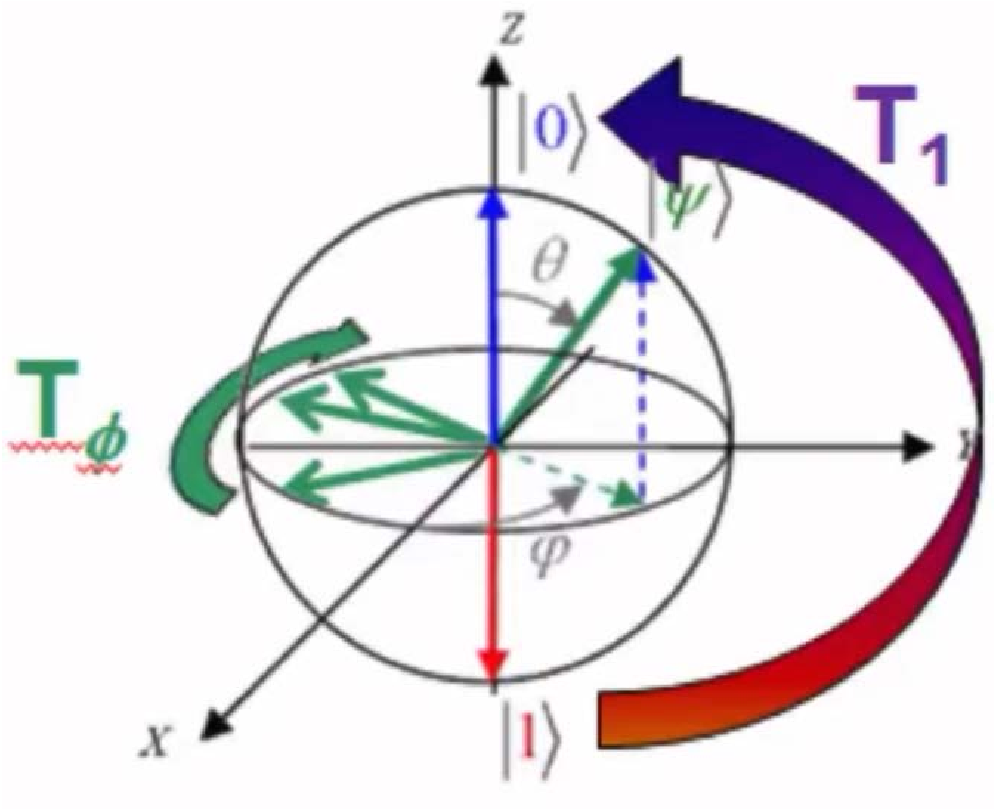
T_2 – Overall Decoherence Time

- Initialize with qubit in the ground state |0>
- Transform the qubit into a superposition state
- Allow the qubit state to evolve over time
- Measure the qubit state (dephasing)

Qubit Decoherence

- Measure a loss of quantum information due to interactions with environmental factors
- T_1 is a relaxation time
- T_ϕ is a dephasing time
- T_2 is the overall decoherence time

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}$$



IBM figure from Device Characteristics presentation

Questions